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# Unsteady, swirling boundary-layer flow with heat and mass transfer in nozzles, diffusers and hydrocyclones

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**Abstract**—The unsteady, laminar, incompressible, swirling boundary-layer flow with heat and mass transfer in a nozzle, a diffuser and a hydrocyclone has been investigated when the free-stream velocity and mass transfer vary with time. The governing non-linear partial differential equations with three independent variables have been solved numerically using an implicit finite-difference scheme. Both the heat transfer and skin friction are found to be strongly affected by swirl and mass transfer, but only moderately affected by the free-stream velocity. However, the Prandtl number strongly affects only the heat transfer, whereas the skin friction is unaffected by it.

## 1. INTRODUCTION

THE ACCURATE prediction of the flow field and heat transfer for unsteady swirling boundary layers in nozzles, diffusers and hydrocyclones is of great importance in many engineering devices such as swirl atomizers, moisture separators, dust collectors, turbines, pumps, compressors and other roto-dynamic machines. In spite of the great importance of the unsteady, swirling flow phenomena in nozzles, diffusers and hydrocyclones, no study has been reported so far. However, the analogous steady case has been studied by several investigators [1–7] using approximate methods. Recently, Meena and Nath [8] and Kumari and Nath [9] have studied the foregoing steady problem using a finite-difference scheme.

It may be remarked that, like nozzles and diffusers which form an important component in many engineering devices mentioned earlier, the hydrocyclone is widely used in industry for separating small dense particles from a liquid of lower density [7, 9]. The flow through a nozzle or a hydrocyclone experiences a favourable pressure gradient whereas the flow through a diffuser encounters an adverse pressure gradient. Hence the flow through a diffuser is likely to separate from the boundary even near the inlet. However, separationless flow in the entire length of the diffuser can be achieved by using appropriate amount of suction.

The aim of the present analysis is to present the flow and heat transfer results for the unsteady laminar swirling incompressible boundary-layer flow in a nozzle, a diffuser and a hydrocyclone when the free-stream velocity and mass transfer vary with time. The non-linear partial differential equations involving three independent variables governing the flow have been solved numerically using an implicit finite-difference scheme [10, 11]. The results have been compared with those available in the literature. It may be remarked that for small swirl at the inlet $(v_s/u_s)_0 \leqslant 5$, the flow will remain laminar in the entire length, however for large swirl $(v_s/u_s)_0 > 5$, it may become turbulent. Here, we have taken $(v_s/u_s)_0 < 3.2$ so that the flow remains laminar in the entire region of nozzle, diffuser and hydrocyclone. For $(v_s/u_s)_0 > 5$, the flow becomes turbulent and the present analysis is not valid for this case. Nevertheless, the present analysis may form the basis of analysis based on more realistic models including turbulent flows.

## 2. GOVERNING EQUATIONS

We consider the unsteady, laminar, incompressible boundary-layer swirling flow through an axisymmetric surface of variable cross-section with imposed vortex at the edge of the boundary layer and mass transfer on the surface. The free-stream velocity and mass transfer are assumed to vary with time. Under the foregoing assumptions the non-linear partial differential equations (with three independent variables) governing the flow can be expressed in dimensionless form as [8, 9]

$$F'' + \phi f F' + \beta\phi(1-F^2) + \alpha\phi(1-s^2) + P_1[\phi^{-1}\phi_{t^*}(1-F) - F_{t^*}] = 2\xi\phi(FF_\xi - f_\xi F') \quad (1a)$$

$$s'' + \phi f s' - P_1[\phi^{-1}\phi_{t^*}s + s_{t^*}] = 2\xi\phi(Fs_\xi - f_\xi s') \quad (1b)$$

$$Pr^{-1} g'' + \phi f g' + (u_s^2/h_e)\phi^2[F'^2 + (v_s/u_s)^2 s'^2] - P_1 g_{t^*} = 2\xi\phi(Fg_\xi - f_\xi g'). \quad (1c)$$

The boundary conditions can be expressed as

$$\begin{aligned} &F(\xi, 0, t^*) = s(\xi, 0, t^*) = 0, g(\xi, 0, t^*) = 0 \\ &F(\xi, \infty, t^*) = s(\xi, \infty, t^*) = g(\xi, \infty, t^*) = 1. \end{aligned} \quad (2)$$

The flow is initially assumed to be steady and then becomes unsteady for $t^* > 0$. Hence, the initial

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## NOMENCLATURE

| | |
|---|---|
| $a, A, b, b_1, B$ | constants |
| $C_f$, $\bar{C}_f$ | skin-friction coefficients in the $x$- and $y$-directions, respectively |
| $C_p$ | specific heat at a constant pressure |
| $f$ | dimensionless stream function |
| $f_w$ | mass transfer parameter |
| $F$ (or $f'$), $s$ | dimensionless longitudinal and tangential (swirl) velocities, respectively |
| $F'_w$, $s'_w$ | surface skin-friction parameters in the $x$- and $y$-directions, respectively |
| $g$ | dimensionless temperature |
| $g_w$ | dimensionless wall temperature |
| $g'_w$ | heat transfer parameter at the wall |
| $h$ | enthalpy |
| $L$ | length of the nozzle or diffuser or hydrocyclone measured in the $x$-direction |
| $Nu$ | Nusselt number |
| $Pr$ | Prandtl number |
| $P_1$ | arbitrary function |
| $r$ | surface or body radius |
| $r_0$ | value of $r$ at $\bar{x} = 0$ |
| $t, t^*$ | dimensional and dimensionless times, respectively |
| $T$ | temperature |
| $u, v, w$ | velocity components in the $x$-, $y$- and $z$-directions, respectively |
| $u_e$, $v_e$ | longitudinal and tangential velocity components at the edge of the boundary layer, respectively |
| $u_s$, $v_s$ | values of $u_e$ and $v_e$ at $t^* = 0$, respectively |
| $u_{s0}$, $v_{s0}$ | values of $u_s$ and $v_s$ at $\bar{x} = 0$ |
| $u_e^2/h_e$ | dissipation parameter |
| $u_s^2/h_e$ | value of $u_e^2/h_e$ at $t^* = 0$ |
| $v_e/u_e$ | ratio of the tangential and longitudinal velocities at the edge of the boundary layer |
| $v_s/u_s$ | value of $v_e/u_e$ at $t^* = 0$ |
| $x, y, z$ | longitudinal, tangential, and normal directions, respectively |
| $\bar{x}$ | dimensionless longitudinal distance. |

Greek symbols

| | |
|---|---|
| $\alpha, \beta$ | swirl and longitudinal acceleration parameters, respectively |
| $\lambda$ | semi-vertical angle of the circular conical nozzle or diffuser or hydrocyclone |
| $\nu$ | kinematic viscosity |
| $\xi, \eta$ | transformed coordinates |
| $\rho$ | density |
| $\tau_x$, $\tau_y$ | wall shear stresses in the $x$- and $y$-directions, respectively |
| $\phi$ | arbitrary function of $t^*$ |
| $\psi$ | dimensional stream function. |

Superscript

| | |
|---|---|
| $'$ | differentiation with respect to $\eta$. |

Subscripts

| | |
|---|---|
| e | denotes conditions at the edge of the boundary layer |
| 0 | denotes inlet conditions |
| s | denotes steady case |
| $t^*$, $\xi$ | denote derivatives with respect to $t^*$ and $\xi$, respectively |
| w | denotes the conditions at the surface or wall. |

conditions for $F$, $s$ and $g$ at $t^* = 0$ are given by the equations governing the steady flow obtained by putting

$$\phi(t^*) = 1, \quad \phi_{t^*} = F_{t^*} = s_{t^*} = g_{t^*} = 0 \tag{3}$$

in equations (1a–c). Here

$$\left.\begin{aligned} \eta = r u_s z/(2\xi)^{1/2}, \quad \xi = \nu \int_0^x u_s r^2 \, dx, \\ t^* = u_{s0} t/L, \quad \psi = (2\xi)^{1/2} f(\xi, \eta, t^*)\phi(t^*), \\ u = r^{-1}\psi_z, \quad w = -r^{-1}\psi_x \end{aligned}\right\} \tag{4a}$$

$$\left.\begin{aligned} u_e = u_s\phi(t^*), \quad v_e = v_s\phi(t^*), \\ u = u_e F, \quad F = f', \quad v = v_e s, \\ (T - T_w)/(T_\infty - T_w) = g, \quad f = \int_0^\eta F \, d\eta + f_w \end{aligned}\right\} \tag{4b}$$

$$\left.\begin{aligned} f_w = -(2\xi)^{-1/2}\phi^{-1}\int_0^x (w)_w r \, dx, \\ h_e = C_p(T_\infty - T_w) \\ \alpha = -(2\xi/r)(dr/d\xi)(v_s/u_s)^2, \\ \beta = (2\xi/u_s)(du_s/d\xi) \end{aligned}\right\} \tag{4c}$$

$$\left.\begin{aligned} v_e/u_e = v_s/u_s, \quad u_e^2/h_e = (u_s^2/h_e)\phi^2, \\ \bar{x} = x/L \quad P_1 = (2\xi u_{s0})/(L\nu r^2 u_s^2). \end{aligned}\right\} \tag{4d}$$

It may be remarked that the vortex (swirl) at the edge of the boundary layer can be obtained by the application of the momentum equation in the $y$-direction at the edge of the boundary layer which is expressed in dimensionless form as

$$(r v_s/r_0 v_{s0})(d\phi/dt^*) + \phi^2(t^*)(u_s/u_{s0}) \times (d/d\bar{x})(r v_s/r_0 v_{s0}) = 0 \tag{5}$$

where the common factor $\rho$ ($\rho \neq 0$) is omitted. The solution of (5) gives

$$v_s/v_{s0} = \left[\exp\left(\int_0^{\bar{x}} A(u_s/u_{s0})^{-1}\,d\bar{x}\right)\right]\Big/(r/r_0) \quad (6a)$$

$$\phi(t^*) = (1+At^*)^{-1}, \quad -A < 1/t^* \quad (6b)$$

where the constant $A$ represents the unsteadiness in the external flow field and $A \gtrless 0$ according to whether the flow is decelerating or accelerating. Thus for the unsteady flow, the swirl at the edge of the boundary layer is given by (6a) which represents a non-free vortex flow. When the flow is steady ($A = 0$), (6a) reduces to $v_s/v_{s0} = (r/r_0)^{-1}$, i.e. $v_s \propto r^{-1}$ which implies that it is a free vortex flow. The equations (1a–c) for $t^* = 0$ reduce to those of steady-state case [1–9]. Also for $t^* = A = \alpha = v_s/u_s = 0$, they reduce to those of non-swirling flows. Furthermore, for $t^* = \xi = \alpha = v_s/u_s = 0$, they reduce to well-known Blasius-type equations. Since our main aim is to study the unsteady swirling flow using a simple model, we have not taken into account the presence of wall using some 'wall functions' as it is not expected to improve the results appreciably.

The skin-friction coefficients in the $x$- and $y$-directions and the heat transfer coefficient in terms of Nusselt number can be expressed in the form

$$C_f = 2\tau_x/\rho u_e^2 = (2/\xi)^{1/2} vr\phi^{-1}F'_w \quad (7a)$$

$$\bar{C}_f = 2\tau_y/\rho u_e^2 = (2/\xi)^{1/2} vr\phi^{-1}(v_s/u_s)s'_w \quad (7b)$$

$$Nu = x(\partial T/\partial z)_w/(T_\infty - T_w) = (vu_s x)(2\xi)^{-1/2} g'_w. \quad (7c)$$

Here the computations have been carried out for a conical nozzle, a diffuser and a hydrocyclone with straight generators. The non-similarity in the flow field is caused by the velocity at the edge of the boundary layer, curvature of the body, and surface mass transfer because they are functions of the longitudinal distance $\bar{x}$. The unsteadiness in the flow field is introduced by the time-dependent free-stream velocity and mass transfer.

For a conical nozzle (with straight generators), we have [8]

$$\left.\begin{aligned} r = L(1-\bar{x})\sin\lambda, \quad \bar{x} = x/L, \quad u_e = u_s\phi(t^*),\\ v_e = v_s\phi(t^*), \quad \phi(t^*) = (1+At^*)^{-1},\\ u_s = b/r^2,\\ u_{s0} = b/r_0^2, \quad r_0 = L\sin\lambda\\ u_s/u_{s0} = (1-\bar{x})^{-2}, \end{aligned}\right\} \quad (8a)$$

$$v_s/v_{s0} = (1-\bar{x})^{-1}\exp\{(A/3)[1-(1-\bar{x})^3]\}, \quad (8b)$$

$$v_e/u_e = v_s/u_s = (1-\bar{x})\exp\{(A/3)[1-(1-\bar{x})^3]\}(v_s u_s^{-1})_0$$

$$\left.\begin{aligned} u_e^2/h_e = (u_s^2/h_e)\phi^2(t^*),\\ u_s^2/h_e = (1-\bar{x})^{-4}(u_s^2/h_e)_0, \quad P_1 = 2\bar{x}(1-\bar{x})^2,\\ \xi = vbx, \quad \xi(\partial/\partial\xi) = \bar{x}(\partial/\partial\bar{x}) \end{aligned}\right| \quad (8c)$$

$$\left.\begin{aligned} \alpha = 2\bar{x}(1-\bar{x})\exp\{(2A/3)[1-(1-\bar{x})^3]\}(v_s/u_s)_0^2\\ \beta = 4\bar{x}(1-\bar{x})^{-1}, \quad f_w = B\bar{x}^{1/2}(1-\bar{x}/2)/\phi(t^*)\\ B = (w)_w L^{3/2}\sin\lambda/(2vb)^{1/2}. \end{aligned}\right| \quad (8d)$$

Similarly, for a conical diffuser (with straight generators), we get [8]

$$r/L = b_1[1+(\bar{x}/b_1)\sin\lambda], \quad b_1 = a/L$$

$$\alpha = -2(\bar{x}/b_1)[1+(\bar{x}/b_1)\sin\lambda]^{-1}\sin\lambda(v_s/u_s)^2 \quad (9a)$$

$$\begin{aligned} v_e/u_e = v_s/u_s = [1+(\bar{x}/b_1)\sin\lambda]\\ \times\exp\{(Ab_1/3\sin\lambda)([1+(\bar{x}/b_1)\\ \times\sin\lambda]^3-1)\}(v_s/u_s)_0 \end{aligned} \quad (9b)$$

$$\left.\begin{aligned} \beta = -4(\bar{x}/b_1)[1+(\bar{x}/b_1)\sin\lambda]^{-1}\sin\lambda,\\ u_s/u_{s0} = [1+(\bar{x}/b_1)\sin\lambda]^{-2},\\ P_1 = 2\bar{x}[1+(\bar{x}/b_1)\sin\lambda]^2 \end{aligned}\right| \quad (9c)$$

$$\left.\begin{aligned} v_s/v_{s0} = [1+(\bar{x}/b_1)\sin\lambda]^{-1}\\ \times\exp\{(Ab_1/3\sin\lambda)([1+(\bar{x}/b_1)\\ \times\sin\lambda]^3-1)\}\\ u_s^2/h_e = [1+(\bar{x}/b_1)\sin\lambda]^{-4}(u_s^2/h_e)_0,\\ B = -(w)_w a^{3/2}/(2vb)^{1/2}\\ f_w = B(\bar{x}/b_1)^{1/2}[1+(\bar{x}/2b_1)\sin\lambda]\phi^{-1}(t^*). \end{aligned}\right\} \quad (9d)$$

Other expressions are the same as in (8).

Also, for a conical hydrocyclone, the flow is similar to that of the nozzle except that the swirl velocity dominates the longitudinal velocity as the vertex of the cone is approached whereas in the nozzle flow it is the other way around [7]. The expressions corresponding to (8) can be written as [7, 9]

$$\left.\begin{aligned} r = L(1-\bar{x})\sin\lambda, \quad u_e = u_s\phi(t^*),\\ v_e = v_s\phi(t^*), \quad u_s = b/r^{1/2},\\ v_s/v_{s0} = (1-\bar{x})^{-1}\exp\{(2A/3)[1-(1-\bar{x})^{3/2}]\} \end{aligned}\right\} \quad (10a)$$

$$\left.\begin{aligned} \alpha = (4/5)(1-\bar{x})^{-7/2}[1-(1-\bar{x})^{5/2}]\\ \times\exp\{(4A/3)[1-(1-\bar{x})^{3/2}]\}(v_s/u_s)_0^2\\ (v_e/u_e)^2 = (v_s/u_s)^2 = (1-\bar{x})^{-1}\\ \times\exp\{(4A/3)[1-(1-\bar{x})^{3/2}\}(v_s/u_s)_0^2] \end{aligned}\right\} \quad (10b)$$

$$\left.\begin{aligned} \beta = (2/5)(1-\bar{x})^{-5/2}[1-(1-\bar{x})^{5/2}]\\ u_s^2/h_e = (1-\bar{x})^{-1}(u_s^2/h_e)_0\\ \xi = (2/5)vbL^{5/2}\sin^{3/2}\lambda[1-(1-\bar{x})^{5/2}]\\ \xi(\partial/\partial\xi) = (2/5)(1-\bar{x})^{-3/2}[1-(1-\bar{x})^{5/2}](\partial/\partial\bar{x}) \end{aligned}\right\} \quad (10c)$$

$$\left.\begin{aligned} P_1 = (4/5)(1-\bar{x})^{-1}[1-(1-\bar{x})^{5/2}]\\ f_w = B\bar{x}(1-\bar{x}/2)[1-(1-\bar{x})^{5/2}]^{-1/2}\phi^{-1}(t^*)\\ B = -(5/4)^{1/2}(w)_w(L^3\sin\lambda)^{1/4}/(vb)^{1/2}. \end{aligned}\right| \quad (10d)$$

The normal velocity at the wall $(w)_w$ is taken here to be constant. Hence, the parameter $B$ (representing surface mass transfer) in equations (8d), (9d), and (10d) is also constant. For flow through a nozzle and a hydrocyclone, one encounters a singularity at $\bar{x} = 1$, because $u_s$, $v_s$, $\beta$, and $u_s^2/h_e$ tend to infinity as $\bar{x}$ tends to 1 [see equations (1), (8) and (10)]. However no singularity is encountered for diffuser flow [see equation (9)].

Therefore, for the flow through a nozzle and a hydrocyclone, the solution is not expected to be valid as $\bar{x}$ tends to 1.

## 3. RESULTS AND DISCUSSION

The set of equations (1) under boundary conditions (2) and initial conditions, obtained by using the conditions given in (3) in (1), and relations (8)–(10) has been solved numerically employing the implicit finite-difference scheme. The description of the method is not presented here as it is described in great detail in refs. [10, 11]. To ensure the convergence of the finite-difference equations to the true solution, the effects of step sizes ($\Delta\eta$, $\Delta\bar{x}$, $\Delta t^*$) on the solution were studied (not shown here in figures to reduce their number) and the optimum values of these step sizes were obtained. Consequently, the computations were carried out with $\Delta\eta = 0.01$, $\Delta\bar{x} = 0.025$ and $\Delta t^* = 0.05$. The edge of the boundary layer $\eta_\infty$ was taken between 4 and 6 depending on the values of the parameters. The results presented here are found to be independent of the step sizes and $\eta_\infty$ at least up to the third decimal place.

In order to assess the accuracy of the present method, the results for the steady flows have been compared with those of [8, 9] and they are found to agree up to the fourth decimal place. The comparison is not presented here for the sake of brevity. Also, we have compared our results with those of unsteady non-similar flow over a rotating sphere. Our governing equations (1) reduce to those of [12] if we replace $\alpha\phi(1-s^2)$ by $\phi^{-1}\phi_1^2\alpha s^2$ in (1a), $\phi f s'$ by $\phi(fs'-\alpha_1 Fs)$ and $\phi^{-1}\phi_{t^*}$ by $\phi^{-1}(\phi_1)_{t^*}$ in (1b), $v_s/u_s$ by $(r\omega/u_s)(\phi^{-1}\phi_1)$ in (1c), and the boundary conditions (2) are modified as follows:

$$s(\xi, 0, t^*) = 1, \quad s(\xi, \infty, t^*) = 0. \tag{11}$$

Appropriate changes have to be made in the expressions for $\alpha, \beta, \xi, f_w$ etc. (see [12]). The comparison has been shown in Fig. 1 and they are found to be in excellent agreement. It was not possible to compare our results with experimental results due to lack of data for the problem considered here. It may be noted that in nozzles and hydrocyclones, the flow is with favourable pressure gradient whereas in diffusers it encounters an adverse pressure gradient. The results for both cases are presented here.

Figures 2(a)–(c) show the effect of the swirl parameter at the inlet $(v_s/u_s)_0$, mass transfer parameter $B$ [injection $(B < 0)$] and Prandtl number $Pr$ on the skin-friction and heat transfer parameters ($F'_w$, $s'_w$, $g'_w$) for nozzle flow. The skin-friction and heat transfer parameters are found to increase with $(v_s/u_s)_0$ and $\bar{x}$. This is because the fluid gets accelerated due to the favourable pressure gradient caused by the nozzle geometry and swirl parameter $(v_s/u_s)_0$. This in turn increases both the skin-friction and heat transfer parameters. Also $F'_w$, $s'_w$ and $g'_w$ increase as $t^*$ increases (in some cases after a certain instant of time). It may be noted that near the inlet $\bar{x} = 0$, the effect of swirl on $F'_w$, $s'_w$ and $g'_w$ is small, because at $\bar{x} = 0$, the parameters associated with the swirl $v_s/u_s$ and $\alpha$ are zero (8). As expected, the skin-friction and heat transfer ($F'_w$, $s'_w$, $g'_w$) are reduced due to injection $(B < 0)$ because injection increases both the momentum and thermal boundary-layer thicknesses. The effect of suction is just the opposite. The heat transfer parameter $g'_w$ is found to increase with the Prandtl number $Pr$ [Fig. 2(c)]. This is to be expected, because a larger Prandtl number results in a thinner thermal boundary layer with a corresponding large temperature gradient at the wall and hence a large surface heat transfer.

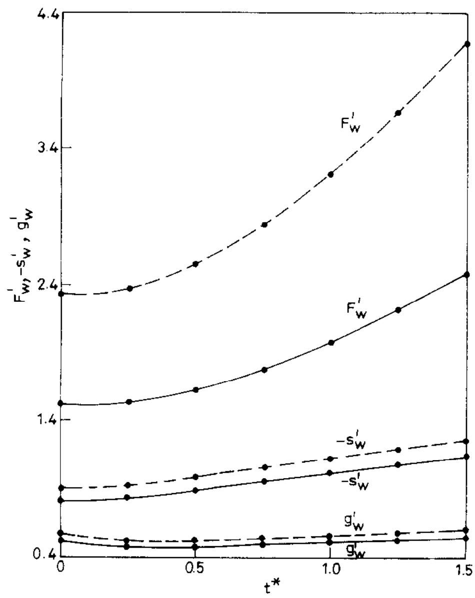


FIG. 1. Comparison of skin friction and heat transfer results with those of Kumari and Nath (circles) for $\phi(t^*) = 1+\varepsilon t^{*2}$, $\phi_1(t^*) = 1+\varepsilon_1 t^{*2}$, $\bar{x} = 0.8$, $u_s^2/h_e = B = 0$, $Pr = 0.7$, $\varepsilon = 0.25$, $\varepsilon_1 = 0.5$.

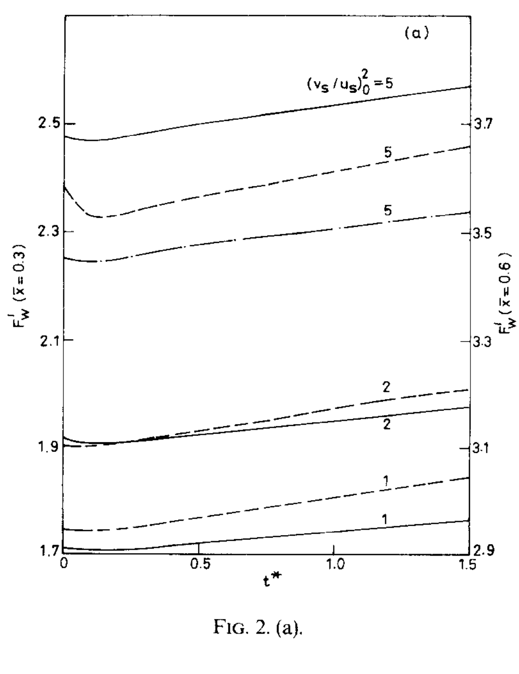


FIG. 2. (a).

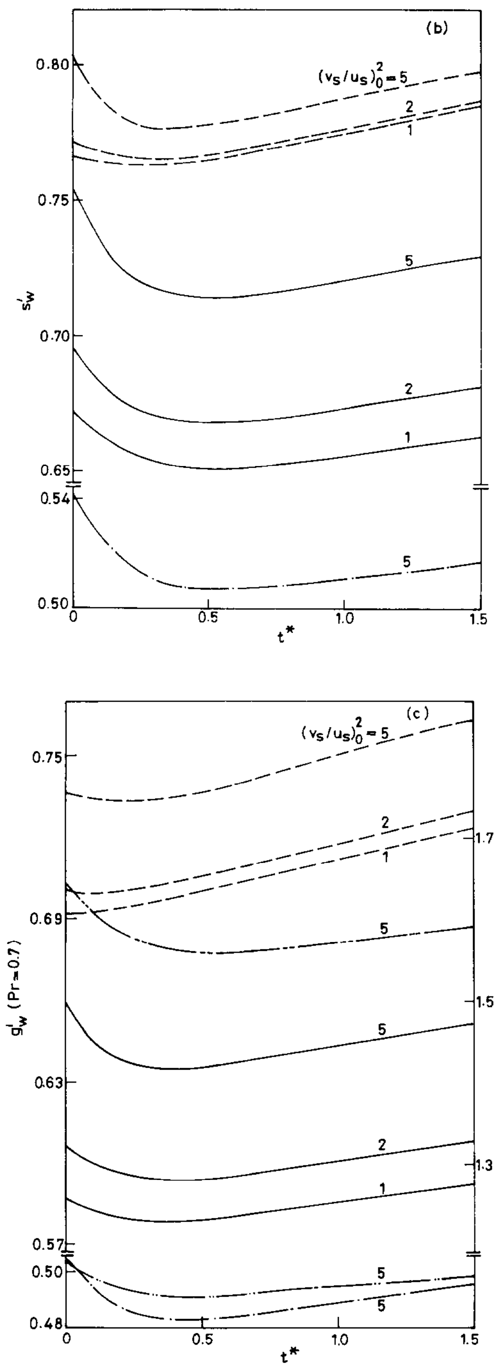


FIG. 2. (a) Skin-friction parameter in the $x$-direction $F'_w$; (b) skin-friction parameter in the $y$-direction $s'_w$. (c) Heat transfer parameter $g'_w$ for $A = -0.05$, $(u_s^2/h_e)_0 = 0.001$: ——, $\bar{x} = 0.3$, $B = 0$, $Pr = 0.7$; – – – –, $\bar{x} = 0.6$, $B = 0$, $Pr = 0.7$; —·—, $\bar{x} = 0.3$, $B = -0.5$, $Pr = 0.7$; —··—, $\bar{x} = 0.3$, $B = 0$, $Pr = 3.0$; –––––, $\bar{x} = 0.3$, $B = 0$, $Pr = 7.0$ (nozzle).

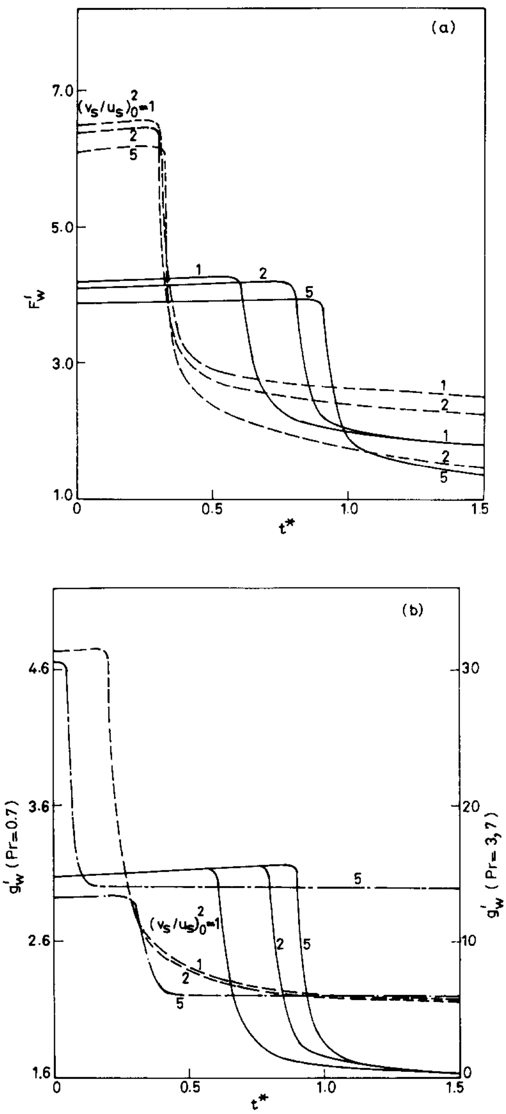


FIG. 3. (a) Skin-friction parameter in the $x$-direction $F'_w$. (b) Heat transfer parameter $g'_w$ for $A = -0.05$, $(u_s^2/h_e)_0 = 0.001$, $b_1 = 0.5$, $B = 2$, $\lambda = 0.1$: ——, $\bar{x} = 0.5$, $Pr = 0.7$; – – – –, $\bar{x} = 1.0$, $Pr = 0.7$; —·—, $\bar{x} = 0.5$, $Pr = 3.0$; –––––, $\bar{x} = 0.5$, $Pr = 7.0$ (diffuser).

The skin-friction and heat transfer parameters $(F'_w, s'_w, g'_w)$ for diffuser flow for some representative values of the parameters have been shown in Figs. 3(a) and (b). Since the behaviour of $s'_w$ is qualitatively same as that of $F'_w$, only results for $F'_w$ are presented. It is observed that separationless flow along the entire length of the diffuser can be obtained by applying appropriate amount of suction which depends on $(v_s/u_s)_0$ and $\lambda$, and it becomes less as $(v_s/u_s)_0$ increases or $\lambda$ decreases. Here, we find that for the values of the parameters used in our calculation the suction parameter $B = 2$ (approximately) is sufficient to ensure the separationless flow along the entire length of the diffuser. We find that both the skin friction and heat transfer $(F'_w, g'_w)$ remain nearly constant up to a certain time $t^*$ (starting from $t^* = 0$) and then rapidly decrease in small intervals of time and finally approach the asymptotic value. This is true for all values of swirl parameter $(v_s/u_s)_0$ and at all longitudinal locations $\bar{x}$. Like nozzle flows, the heat transfer parameter $g'_w$

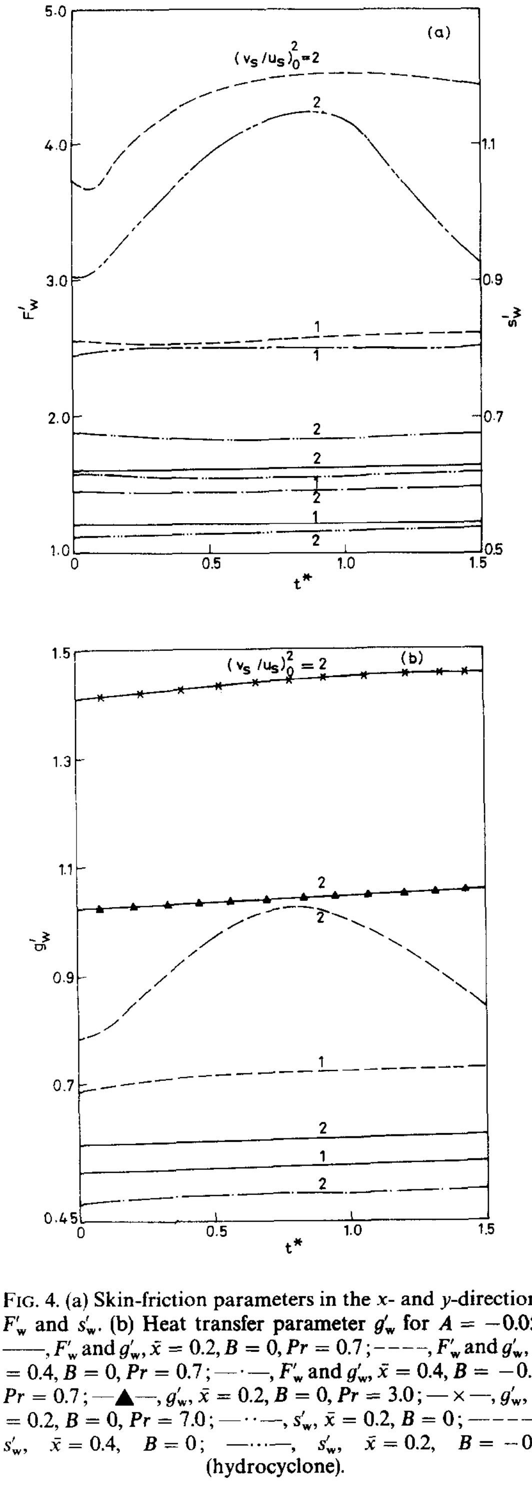


FIG. 4. (a) Skin-friction parameters in the $x$- and $y$-directions $F'_w$ and $s'_w$. (b) Heat transfer parameter $g'_w$ for $A = -0.05$: ——, $F'_w$ and $g'_w$, $\bar{x} = 0.2, B = 0, Pr = 0.7$; ----, $F'_w$ and $g'_w$, $\bar{x} = 0.4, B = 0, Pr = 0.7$; —·—, $F'_w$ and $g'_w$, $\bar{x} = 0.4, B = -0.5, Pr = 0.7$; —▲—, $g'_w$, $\bar{x} = 0.2, B = 0, Pr = 3.0$; —×—, $g'_w$, $\bar{x} = 0.2, B = 0, Pr = 7.0$; —··—, $s'_w$, $\bar{x} = 0.2, B = 0$; ----, $s'_w$, $\bar{x} = 0.4, B = 0$; —···—, $s'_w$, $\bar{x} = 0.2, B = -0.5$ (hydrocyclone).

increases with $Pr$. However, the effect is much more pronounced as compared to the nozzle case.

The skin-friction and heat transfer results ($F'_w, s'_w, g'_w$) for the flow through the hydrocyclone are presented in Figs. 4(a) and (b) which show the effects of the swirl parameter, mass transfer (injection), Prandtl number (only on $g'_w$) and longitudinal distance $\bar{x}$. It is observed that the effect of swirl $(v_s/u_s)_0$, injection ($B < 0$), and $Pr$ (on $g'_w$ only) on $F'_w, s'_w$ and $g'_w$ is much more pronounced than that of time $t^*$ which is similar to that observed for the nozzle case. It may be noted that in the flow through the hydrocyclone, the swirl flow dominates the longitudinal flow which increases with $\bar{x}$. Hence in order to ensure the laminar flow in the entire region, we have taken $(v_s/u_s)_0 < 2$.

## 4. CONCLUSIONS

For unsteady swirling flows, the swirl velocity at the edge of the boundary layer is of non-free vortex type in contrast to the steady flow where it is of free vortex type. Also, the longitudinal and swirl velocities at the edge of the boundary layer vary inversely as a linear function of time. The skin-friction and heat transfer are strongly affected by the swirl velocity, mass transfer, and Prandtl number ($Pr$ affects only $g'_w$) whereas they are weakly dependent on time $t^*$, except for diffusers. Separationless flow along the entire length of the diffuser can be obtained by applying appropriate amount of suction.

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## ECOULEMENT VARIABLE, TOURBILLONNAIRE A COUCHE LIMITE AVEC TRANSFERT DE CHALEUR ET DE MASSE DANS DES TUYERES, DIFFUSEURS ET HYDROCYCLONES

**Résumé**—L'écoulement variable, tourbillonnaire, laminaire, à couche limite avec transfert de chaleur et de masse dans une tuyère, un diffuseur et un hydrocyclone est étudié lorsque la vitesse d'écoulement libre et le transfert massique varient avec le temps. Les équations aux dérivées partielles, non linéaires, avec trois variables indépendantes sont résolues numériquement en utilisant un schéma implicite aux différences finies. Le transfert de chaleur et le frottement pariétal sont tous les deux fortement affectés par le tourbillon et le transfert de masse, mais faiblement affectés par la vitesse d'écoulement libre. Néanmoins le nombre de Prandtl influe fortement sur le transfert thermique seul, tandis que le frottement pariétal lui est insensible.

## INSTATIONÄRE WIRBELGRENZSCHICHTSTRÖMUNG MIT WÄRME- UND STOFFÜBERGANG IN DÜSEN, DIFFUSOREN UND HYDROCYCLONEN

**Zusammenfassung**—Die instationäre, laminare und inkompressible Wirbelgrenzschichtströmung mit Wärme- und Stoffübergang wurde bei zeitlich sich ändernden Werten von Freistrahl-Geschwindigkeit und Stoffübergang in einer Düse, einem Diffusor und einem Hydrocyclon untersucht. Die bestimmenden nichtlinearen partiellen Differentialgleichungen mit drei unabhängigen Variablen werden numerisch mit Hilfe eines impliziten Finite–Differenzen-Verfahrens gelöst. Sowohl der Wärmetransport als auch die Wandreibung werden stark vom Wirbel- und Stofftransport beeinflußt, dagegen aber nur wenig von der Freistrahl-Geschwindigkeit. Die Prandtl-Zahl dagegen beeinflußt den Wärmetransport stark, die Wandreibung jedoch nicht.

## ТЕПЛО-И МАССООБМЕН В НЕСТАЦИОНАРНОМ ЗАКРУЧЕННОМ СЛОЕ В СОПЛАХ, ДИФФУЗОРАХ И ГИДРОЦИКЛОНАХ

**Аннотация**—Исследован тепло-и массоперенос в нестационарном ламинарном несжимаемом закрученном погранслое в сопле, диффузоре и гидроциклоне, когда скорость свободного потока и массоперенос меняются со временем. Определяющие нелинейные уравнения в частных производных с тремя независимыми переменными решались численно с использованием неявной конечно-разностной схемы. Обнаружено, что теплоперенос и поверхностное трение сильно зависят от степени закрутки и массопереноса, тогда как скорость свободного потока влияет на них незначительно. Вместе с тем, число Прандтля сильно влияет только на теплоперенос, а поверхностное трение от него не зависит.